Data you should know/memorize: (will NOT be on formula sheet!) **data since midterm in blue**

Water molecule D_{water}=0.3 nm; V_{water}=0.03 $(nm)^3$; ρ_{water}=1 g/cm^3=10^3 kg/m^3

E. coli: dimensions=2 μm x 1 μm; $V_{E. coli}$=1 $(\mu m)^3$; $A_{E. coli}$=6 $(\mu m)^2$; $m_{E.coli}$= 1 pg; generation~3000 s; $N_{protein}$=2.4x10^6; N_{carbon}=10^{10}.

S. cerevisiae: D_{yeast}=5 μm; V_{yeast}=60 $(\mu m)^3$; A_{yeast}=75 $(\mu m)^2$; m_{yeast}=60 pg

Typical amino acid mass = 100 Da; Typical number of amino acids/protein = 300

Genome sizes: E. coli 5x10^6 bp; human 3x10^9 bp; DNA: length~1/3 nm/bp; volume~1 nm^3/bp

Human basal metabolic rate Γ_0=100 W

Dielectric constant of water: D_{water}=80

Lipid bilayer thickness= 4nm; headgroup area=0.5 nm^2

Hydrophobic surface energy: $\sigma \approx 10\, k_B T/nm^2 = 0.04\, J/m^2$

Reynolds number $R_e \equiv \frac{\rho v \ell}{\eta}$: turbulent onset R_e~1000—2000; bacterial R_e~10^{-6}

Formula sheet:

$$P_{heat} = \kappa_T \frac{S\Delta T}{d};\ P_{rad} = \varepsilon\sigma T^4; Q = mC\Delta T; \Gamma_0(M) \approx aM^{3/4}\ (a \sim 4)$$

Probability: $\int_{-\infty}^{\infty} dx\, P(x) = 1;\ \langle f(x)\rangle_P \equiv \int_{-\infty}^{\infty} dx\, P(x) f(x);\ \sigma^2 = \left\langle \left(x - \langle x\rangle\right)^2 \right\rangle = \left\langle x^2 \right\rangle - \langle x\rangle^2$

$$P_{Gaussian}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x_0)^2}{2\sigma^2}},\ P_{N,M} \approx \frac{1}{\sqrt{2\pi Npq}} e^{-\frac{(M-\langle M\rangle)^2}{8NpqN}} \text{ (discrete)}$$

Random walks: $P_N(X) = \frac{1}{\sqrt{2\pi Na^2}} e^{-\frac{X^2}{2Na^2}}$ (1D); $P_N(\vec{R}) = \left(\frac{3}{2\pi Na^2}\right)^{3/2} e^{-\frac{3R^2}{2Na^2}}$ (3D); $\frac{1}{D}\frac{\partial P}{\partial t} = \nabla^2 P; \left\langle R^2 \right\rangle = 2dDt$;

$$P(X,t) = \frac{1}{\sqrt{4\pi Dt}} e^{-\frac{X^2}{4Dt}} \text{ (1D)};\ P(\vec{R},t) = \left(\frac{1}{4\pi Dt}\right)^{3/2} e^{-\frac{1}{4Dt}\left(X^2+Y^2+Z^2\right)} = \left(\frac{1}{4\pi Dt}\right)^{3/2} e^{-\frac{R^2}{4Dt}} \text{ (3D)}$$

Thermodynamics and Stat Mech: $đQ = dE + đW$; $TdS = dE + PdV - \mu dN$; $E - TS + PV - \mu N = 0$;

$$VdP = SdT + Nd\mu;\ \langle E\rangle_c = \frac{\partial}{\partial\beta}\ln Z, \beta = \frac{1}{k_B T};\ \left.\frac{\partial E}{\partial T}\right|_{V,N} \equiv C_{V,N} = \frac{1}{k_B T^2}\left\langle \left(E - \langle E\rangle_c\right)^2 \right\rangle_c$$

1. Microcanonical ensemble: System isolated. All states between E and E+ΔE weighted equally.

$$P_n = \frac{1}{W(E,V,N,(\Delta E))} = \frac{1}{\Omega(E,V,N)\Delta E}; S(E,V,N) = k_B \ln W, k_B \ln\Omega;\ dS = \frac{1}{T}dE + \frac{P}{T}dV - \frac{\mu}{T}dN$$

Isolated system maximizes entropy.

2. Canonical ensemble: System in contact with heat bath at temperature T.

$$P_n = \frac{1}{Z} e^{-\frac{E_n}{k_B T}}, Z(T,V,N) = \sum_n e^{-\frac{E_n}{k_B T}}; F(T,V,N) = -kT\ln Z = E - TS;\ dF = -SdT - PdV + \mu dN$$

System minimizes F at fixed T.

3. Grand (canonical) ensemble: System in contact with heat bath T and particle reservoir μ.

$$P_n = \frac{1}{\Xi} e^{-\frac{1}{k_B T}(E_n - \mu N_n)},\ \Xi(T,V,\mu) = \sum_N e^{\frac{\mu N}{k_B T}} Z(T,V,N); \Psi(T,V,\mu) = -k_B T\ln\Xi = -PV$$

System minimizes Ψ at fixed T and μ.

4. Constant pressure ensemble: System in contact with heat bath T and subject to constant pressure P.

$P_n = \frac{1}{Z} e^{-\frac{1}{k_B T}(E_n + PV_n)}$, $Z(T,P,N) = \int_V dV e^{-\frac{PV}{k_B T}} Z(T,V,N); G(T,P,N) = -k_B T \ln Z = \mu N \;\; dG = -SdT + VdP + \mu dN$

System minimizes G at fixed T and P.

Ideal Monatomic Gas: $PV = Nk_BT$; $E = \frac{3}{2} N k_B T$; $\sum_n \to \frac{1}{N!} \int_V \prod_{k=1}^{N} \frac{(d^3 p_k)(d^3 r_k)}{(2\pi\hbar)^{3N}}$; $n = \frac{e^{\frac{\mu}{k_B T}}}{\lambda_{th}^3}$;

$\frac{1}{\lambda_{th}} = \int_{-\infty}^{\infty} \frac{dp}{2\pi\hbar} e^{-\frac{p^2}{2mk_BT}} = \sqrt{\frac{mk_BT}{2\pi\hbar^2}}$; $S((E,V,N) = Nk_B\left[\ln\left(\frac{E^{3/2}V}{N^{5/2}}\right) + \frac{3}{2}\ln\left(\frac{m}{3\pi\hbar^2}\right) + \frac{5}{2}\right]$; $Z(T,V,N) = \frac{1}{N!}\left(\frac{V}{\lambda_{th}^3}\right)$

Chemical reactions/ideal solns:

$\nu_A \mu_A + \nu_B \mu_B = \nu_C \mu_C + \nu_D \mu_D$; $n = \frac{z(T) e^{-\frac{\varepsilon_s}{k_B T}} \cdot e^{\frac{\mu}{k_B T}}}{\lambda_{th}^3}$; $\frac{n_C^{\nu_C} n_D^{\nu_D}}{n_A^{\nu_A} n_B^{\nu_B}} = K_{eq}$; $P_{os} = c_s k_B T$; $\log_{10} \frac{[M^-]}{[HM]} = pH - pK$

Polymers: $\langle R^2 \rangle = \ell_K L$; $\langle R^2 \rangle = \ell_K L$; $\xi_p \sim \frac{\kappa}{k_B T}$; $\langle R^2 \rangle = 2\xi_p L$; $\ell_K = 2\xi_p \sim \frac{2\kappa}{k_B T}$; $\langle R^2 \rangle \sim a_{eff}^2 N^{2\nu}$

$E = \frac{\kappa}{2} \int_0^L ds \frac{1}{R^2(s)}$; $F_{coil} \approx -TS = -Nk_BT \ln z$; $F_{compact} \approx E = -|\varepsilon| N$; $k_{spring} = \frac{dk_BT}{\langle R^2 \rangle}$; $R_G^2 = \frac{1}{N} \sum_{n=1}^{N} \left\langle \left(\vec{R} - \vec{R}_{CofM}\right)^2 \right\rangle$

Electrostatic effects: $F = \frac{|q_1 q_2|}{4\pi\varepsilon r^2}$; $\vec{E} = -\vec{\nabla}\Phi$; (Gauss) $Flux = \frac{q}{\varepsilon}$; $\nabla^2 \Phi(\vec{r}) = -\frac{\rho(\vec{r})}{\varepsilon} = -\frac{1}{\varepsilon} \sum_\alpha q_\alpha n_\alpha^0 e^{-\frac{q_\alpha \Phi(\vec{r})}{k_B T}}$;

$\frac{1}{\lambda_D^2} = \frac{1}{\varepsilon k_B T}\left(\sum_\alpha q_\alpha^2 n_\alpha^0\right)$; $\ell_B \equiv \frac{q^2}{4\pi\varepsilon k_B T}$;

Membranes: $K_s \approx 4\sigma$; $\kappa_b = 25\ k_BT \sim K_s d^2$; $\xi_p \sim d e^{\frac{4\pi}{3} \cdot \frac{\kappa_b}{k_B T}}$; $E_b = \frac{\kappa_b}{2} \int_S dA \left(\frac{1}{R_1} + \frac{1}{R_2} - C_0\right)^2$

Hydrodynamics: $\frac{\partial n}{\partial t} = -\vec{\nabla} \cdot \vec{j}$; $\vec{\nabla} \cdot \vec{v} = 0$; $\rho \frac{\partial \vec{v}}{\partial t} + \rho(\vec{v} \cdot \vec{\nabla})\vec{v} = -\vec{\nabla} P + \eta \nabla^2 \vec{v} + \vec{f}$;

$F = \eta A \frac{dv}{dz}$; $F_{drag} = 6\pi\eta R v$; $Q = \frac{\pi}{8} \cdot \frac{R^4}{\eta} \cdot \frac{\Delta P}{L}$; $ma = -\gamma v + f$; $v_d = \frac{f}{\gamma}$

Constants: $\sigma = 5.67\text{x}10^{-8}$ W/m^{2}K^4, $k_B = 1.38\text{x}10^{-23}$ J/K, $N_A = 6.02\text{x}10^{23}$; 1 amu=$1.67\text{x}10^{-27}$ kg; 1 cal = 4.185 J; T_{room}=300 K; $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/Jm;

Math: $I_n(a) = \int_0^\infty dx\, x^n e^{-ax^2}$; $I_0(a) = \frac{1}{2}\sqrt{\frac{\pi}{a}}$; $I_1(a) = \frac{1}{2a}$; $\frac{d}{da} I_n(a) = -I_{n+2}(a)$

$N! = \sqrt{2\pi N}\, N^N e^{-N}\left(1 + \frac{1}{12N} + \ldots\right)$